StrutTie Design Example

Design of deep beams subjected to concentrated load
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1.5 Summary
Part 1: Introduction

1. Strut-tie model approach

In general, structural concrete can be classified into B-regions (beam or Bernoulli regions) and D-regions (disturbed or discontinuity regions). B-regions are the parts of structure where the assumption of the linear strain distribution of theory of flexure applies. D-regions are another parts of structure in which the strain distribution is highly nonlinear due to concentrated load, reaction, and geometrical discontinuity. Most design practices for B-regions are well carried out by applying the traditional theory of flexure and parallel chord truss analogy for shear. On the other hand, design practices for D-regions are mostly based on empirical approaches depending on experimental results and past experience. For most types of D-regions, codes provide little guidance to designers. Because of the absence of rational code provisions for D-regions, most structural problems occur in D-regions.

The strut-tie model approach, a method for design of structural concrete, is proposed to improve the problems of current design codes. A strut-tie model, which is made up of struts and ties connected at nodes, allows a rational design of structural concrete subject to complex loading and geometrical conditions. Also, the model promotes a better understanding of load transfer mechanisms and structural behavior and it improves the designers' abilities to handle unusual circumstances. These advantages have originated the selection of the strut-tie model approach in major design codes (ACI 318, AASHTO LRFD, CSA, DIN, Eurocode 2, FIB).

When considering the strut-tie model approach from a practical viewpoint, however, there are several uncertainties. First, the difference of strut-tie model design results may be acquired under the same circumstances, as a strut-tie model is generally selected based on the designer's experience and subjectivity in the current load path (elastic stress trajectories) method. Reliability on design results may be greatly reduced if a designer is deficient in understanding the structural behavior and load transfer mechanism. To overcome this problem, studies on the construction of proper strut-tie models using performance-based and topology-based optimization techniques have been conducted. However, the construction of strut-tie models by using the optimization techniques has the problem that a fine finite element (FE) mesh generation is required for initial FE modeling, demanding much time and effort in practice.

Second, idealizing a strut-tie model as a simple determinate truss structure for the simplicity of structural analysis causes difficulties in reflecting the actual structural behavior in design by simplifying complex internal stress flows. In addition, simple determinate strut-tie models limit the effective reinforcement details by inducing difficulties in terms of practical horizontal and/or vertical
placements of reinforcing bars and congestion of reinforcing bars due to concentrated tensile forces in steel ties. This problem can be overcome by using indeterminate strut-tie models. However, analysis of indeterminate strut-tie model has the disadvantage that must be accompanied by numerical methods.

Third, the design procedure of the structural concrete having multiple load cases and load combinations is inefficient and time consuming in practical aspect because each strut-tie model must be prepared to handle each different loading situation. Also, a practical strut-tie model design requiring time consuming design process such as hand-based design process and many geometric detailing process is complicated. These problems interrupt the strut-tie model approaches to be widely used as the practical design method for structural concrete.

2. AStrutTie: the automatic design software for strut-tie model method

The strut-tie model approach has been recognized as an efficient methodology for the design of all types of concrete members with D-regions, and the approach has been accepted in design codes globally. However, the design of concrete members with the approach requires many iterative numerical structural analyses, numerous graphical calculations, enormous times and efforts, and designer’s subjective decisions in terms of the development of appropriate strut-tie model, determination of required areas of struts and ties, and verification of strength conditions of struts and nodal zones.

HanGil IT Co., LTD. has developed the AStrutTie, the automatic design software for strut-tie model approach, that enables the design of concrete members efficiently and professionally by overcoming the aforementioned limitations of the strut-tie model approach. In the AStrutTie, the numerical programs that are essential in the strut-tie model analysis and design of concrete members including finite element analysis programs for the plane truss and solid problems with all kinds of boundary conditions, a program for automatic determination of effective strengths of struts and nodal zones, and a program for graphical verification of the appropriateness of developed strut-tie models by displaying various geometrical shapes of struts and nodal zones, are loaded. Also, the numerical program that constructs the proper strut-tie models using evolutionary structural optimization techniques, is loaded. Great efficiency and convenience during the application of the strut-tie model approach may be provided by the various graphics environment-based functions of the software.
3. Aims and contents of this publication

As mentioned above, the strut-tie model approach is emerging as a rational, consistent, and code-worthy methodology for the design of D-regions in structural concrete. Also, this approach has already been adopted as a design method in many design codes. However, the strut-tie model approach has limitations in practical design applications since it require a lot of time and effort without a professional software for the approach. This publication suggests an alternative that solves the practical problems of strut-tie model approach by introducing the sample problems designed with the AStrutTie.

AStrutTie has the following principal features:

- Pre- and post-process for finite element analysis of 2-dimensional solid and truss problems
- Finite element analysis of 2-dimensional solid and truss problems subject to various kinds of complicated boundary conditions
- ESO (Evolutionary Structural Optimization) for construction of 2-dimensional strut-tie models
- Automatic calculation of load carrying capacities of struts and ties (required cross-sectional areas)
- Construction of dimensioned shapes of concrete struts, ties, and nodal zones under ultimate design loads
- Nine strut-tie model templates for convenient, fast, and effective designs of 2-dimensional concrete members
- Automatic creation of structural design reports

This publication contains two parts. After the introduction (Part 1), Part 2 (the major part of this publication) presents ten examples designed with the automatic design software. Most of the examples were taken from other references.
Part II : Design Examples

1. Design of deep beams subjected to concentrated load

1.1 Introduction

The strut-tie model design of simply supported deep beams is usually conducted by the determinate strut-tie models representing an arch mechanism shown in Fig. II-1-1(a), or a truss mechanism shown in Fig. II-1-1(b). The cross-sectional forces of struts and ties in these types of strut-tie models are determined regardless of the stiffness of struts and ties.

Fig. II-1-1 Typical strut-tie models for reinforced concrete deep beams
The CSA (2004) and AASHTO LRFD (2014) have suggested a basic concept of a strut-tie model that satisfies equilibrium and constitutive relationships, and they have allowed the design of reinforced concrete deep beams with the strut-tie model shown in Fig. II-1-1(a). This has influenced the ACI 318-14 (2014) to allow the same model for the reinforced concrete deep beams with the requirement that the angle between a concrete strut and a tie be greater than 25 degrees. When the requirement on the angle is considered, the strut-tie model shown in Fig. II-1-1(a) is used for the beams with a shear span-to-effective depth ratio a/d of less than 1.93 (a/z ≤2.14, z = 0.9d). In addition, according to the design book of the ACI Subcommittee 445-1 (2002), the reinforced concrete deep beams with a/d of larger than 1.93 can be designed by using the strut-tie model shown in Fig. II-1-1(b).

FIB (2010) suggested the determinate and indeterminate strut-tie models of Figs. II-1-1(a), 1(b), and 1(c) for reinforced concrete deep beams, representing respectively an arch load transfer mechanism for a/z ≤0.5, a truss load transfer mechanism for a/z ≥2.0, and a combination of arch and truss load transfer mechanisms for 0.5 < a/z < 2.0. As the strut-tie model in Fig. II-1-1(c) is the first-order indeterminate truss structure, a load distribution ratio was proposed to calculate the cross-sectional forces of struts and ties by simply employing the force equilibrium equations at nodes. With the proposed load distribution ratio $\bar{U}$ varying linearly as a function of a/z as shown in Eqn. (II-1-1), the cross-sectional force of a vertical steel tie $P_w$ in the truss mechanism of Fig. II-1-1(a) is directly obtained from the following equation:

$$\bar{U} = P_w / P = (2a/z - 1) / 3 \quad (\text{II-1-1})$$

In the following sections, the deep beam introduced in ACI Subcommittee 445-1 (2002) is designed by using the three types of aforementioned strut-tie models. The ACI 318M-14 code and the software AStrutTie was used in the design.

1.2 Design example - strut-tie model representing arch mechanism

1.2.1 Problem statement

Design the simply supported deep beam that carries two concentrated factored loads of 953.6 kN (= 1.6 × 596 kN) each on a clear span of 3.66 m, as shown in Fig. II-1-2. The deep beam has a thickness of 356 mm and a 1,220 mm overall depth. The length of the bearing plate at each concentrated load location is 406 mm and the width is the same as the beam thickness, i.e. 356 mm. Use $f_{ck} = 27.6$ MPa and $f_y = 414$ MPa. Neglect the self-weight.
For the deep beam designs, a template provided in the AStrutTie can be used to make the designs easier. The template consists of five input tabs: Dimension, Bearing, Load, STM setting, Rebar. In the Dimension tab, the dimensions of the deep beam are typed in, as shown below.

In the Bearing tab, the information on the bearing plates including the number of bearing plates, locations, width, depth, and thickness are typed in, as shown below.
In the Load tab, the number of load types (dead load, live load, wind load, earthquake load, etc) and the magnitude of service loads are determined. The magnitude of loads can be modified in the later process.

In the STM Setting tab, the locations of the top and bottom horizontal elements of the deep beam strut-tie model are assigned. The locations of the top and bottom elements are usually determined by considering the depth of the equivalent stress block and the effective depth of beam for flexure. In this example, the elements are placed 127 mm and 102 mm away from the top and bottom surfaces of the deep beam, respectively.
In the Rebar tab, the information on the reinforcing bars and details is assigned. The assigned information can be altered freely in the later process.

The 'Beginning Mode' is activated by clicking 'OK' button in the above window. All the entered information can be examined or modified in the Work Tree Window.
1.2.2 Construction of strut-tie model

To conduct the strut-tie model design of the deep beam, the ‘Beginning Mode’ must be switched to the ‘Modeling Mode’. The Modeling Mode consists of ESO, Stress Flow, Truss. As the template for deep beam is provided in the software, the most appropriate strut-tie model is constructed automatically. In this example, however, the finite element analyses for ESO and Stress Flow are carried out to illustrate the procedure for predicting the load transfer mechanisms of the deep beam.

Before conducting the finite element analyses for ESO and Stress Flow, the load factor for live load, 1.6, is assigned in the ‘DEFINE-Load Combinations’.
The finite element analysis results are shown below.

(a) Evolutionary structural optimization 

(b) Principal compressive stress flow

Fig. II-1-3 Analysis results of ESO & Stress Flow

The strut-tie model for the deep beam is constructed by considering the principal stress flow and practical reinforcement patterns. When the template for deep beam is used, the strut-tie model
representing a combined load transfer mechanism as shown in Fig. II-1-1(c) is selected automatically as the shear span-to-effective depth ratio of the deep beam is 1.27. In this example, the strut-tie model representing an arch load transfer mechanism as shown in Fig. II-1-4 is constructed for design. The top and bottom horizontal elements are placed 127 mm and 102 mm away from the top and bottom surfaces of the deep beam, respectively.

![Constructed strut-tie model](image)

**Fig. II-1-4 Constructed strut-tie model**

### 1.2.3 Analysis of strut-tie model

The cross-sectional forces of struts and ties are determined by carrying out the finite element analysis of the constructed strut-tie model.
1.2.4 Strength verification and required rebars

1.2.4.1 Strength under bearing plates

The strength conditions under bearing plates are verified by examining the following equation.

\[ \bar{\phi}_{fb} = \bar{\phi} 0.85 \beta_n f_{ck} O_F / bw = \bar{\phi}_b \]  

(II-1-2)

where, \( \bar{\phi} \) = strength reduction factor of nodal zone (\( = 0.75 \) )

\( \bar{\phi}_n \) = effective strength coefficient of nodal zone (CCC = 1.0, CCT = 0.8, CTT = 0.6)

\( f_{ck} \) = compressive strength of concrete (\( = 27.6 \) MPa)

\( F_u \) = ultimate load acting on bearing plate

\( w \) = length of the bearing plate

\( b \) = width of the bearing plate
### 1.2.4.2 Required area of rebars

The required areas of main reinforcing bars are determined by the following equation. The requirement on the main reinforcing bars is examined in the Design Review as shown below.

\[ A_{s,req} = \frac{F_u}{\phi f_y} \]  

where, \( \phi = \) strength reduction factor of steel tie (= 0.75) 
\( F_u = \) cross-sectional force of steel tie 
\( f_y = \) yield strength of steel (= 414 MPa)

As shown above, the provided area of reinforcing bars (8-D25, 4,077mm²) is not enough to carry the cross-sectional force of steel tie. Thus, 2-D19 bars are added in the Define-Reinforcement Ties.
After additional placement of reinforcing bars, the requirement on the main reinforcing bars is satisfied as shown below.

1.2.4.3 Strength verification of struts

The strength condition of a concrete strut is verified by comparing the required width with provided width of the concrete strut, as shown below.

\[
\frac{w_{\text{req}}}{w_{\text{prov}}} = \frac{F_u}{(0.85\delta_0 f_{ck}) \cdot \beta w_{\text{prov}}} \leq (I - 1 - 4) \tag{II-1.4}
\]

where, \(\delta_0\) = strength reduction factor of concrete strut (= 0.75)

\(\beta_0\) = effective strength coefficient of concrete strut

\(f_{ck}\) = compressive strength of concrete (= 27.6 MPa)
b = width of the beam

The strength conditions of concrete struts can also be verified visually as shown below.

Fig. II-1-6 Required/proposed area of concrete strut
1.2.4.4 Strength verification of nodal zones

The strength condition of a nodal zone is verified by comparing the required width with the provided width of the nodal zone boundary, as shown below.

\[ w_{\text{req}} = \frac{F_u}{(\tilde{\phi} 0.85 \beta_n f_{\text{ck}} b) \hat{w}_{\text{prov}}} \]  

(II-1-5)

where, \( \tilde{\phi} = \) strength reduction factor of nodal zone (= 0.75)

\( \beta_n = \) effective strength coefficient of nodal zone (CCC = 1.0, CCT = 0.8, CTT =0.6)

\( f_{\text{ck}} = \) compressive strength of concrete (= 27.6 MPa)

\( b = \) width of the beam

As shown in the above table, the strength condition about the nodes 3 and 4 is not satisfied. To satisfy the strength condition, the available width of horizontal lower tie must be increased by moving the tie upwards.

1.2.5 Minimum rebars for crack control

Since the effective strength coefficient 0.75 was taken for the two inclined struts of the strut-tie model, the ACI 318M-14 requirement for minimum reinforcing bars for crack control must be satisfied.

\[ \sum \left( \frac{A_{s_i}}{b_s} \right) \sin \alpha_i \hat{\phi} 0.003 \]  

(II-1-6)

where \( A_{s_i} \) is the total area of distributed reinforcement at spacing \( s_i \) in the i-th direction of reinforcement crossing a strut at an angle \( \hat{\phi} \) to the axis of a strut, and \( b_s \) is the width of the strut.
As shown in the above table, the assigned amounts of reinforcing bars are well above the required. The reinforcing bars can be reduced in the Define-Reinforcement Ties menu.

1.3 Design example - strut-tie model representing vertical truss mechanism

1.3.1 Problem statement

In this section, the deep beam is designed by the strut-tie model representing the vertical truss load transfer mechanism shown in Fig. II-1-1(b). As illustrated in the previous sections, the dimensions and the information on the bearing plates and loads are determined in the same way.

1.3.2 Construction of strut-tie model

The Beginning Mode is switched to the Modeling Mode to construct a strut-tie model for the deep beam. As the shear span-to-effective depth ratio of the deep beam is 1.27, the indeterminate strut-tie
model that represents a combined load transfer mechanism is selected automatically from the template for deep beam. In this example, the strut-tie model representing an vertical truss load transfer mechanism as shown in Fig. II-1-7 is constructed for design. The top and bottom horizontal elements are placed 127 mm and 102 mm away from the top and bottom surfaces of the deep beam, respectively.

Fig. II-1-7 Constructed strut-tie model

1.3.3 Analysis of strut-tie model

The cross-sectional forces of struts and ties are determined by carrying out the finite element analysis of the constructed strut-tie model.
1.3.4 Strength verification and required rebars

1.3.4.1 Strength under bearing plates

The strength conditions under bearing plates are examined by the method illustrated in Section 1.2.4.1.

1.3.4.2 Required area of rebars

The required areas of main reinforcing bars are determined by the following equation. The requirement on the reinforcing bars is examined in the Design Review as shown below.

\[ A_{s, \text{req}} = \frac{F_u}{\bar{\mu} f_y} \]  \hspace{1cm} (II-1-7)

where, \( \bar{\mu} \) = strength reduction factor of steel tie (= 0.75)

\( F_u = \) cross-sectional force of steel tie
f_y = yield strength of steel (= 414 MPa)

The spacing of shear reinforcing bars is determined by the following equation.

\[ \bar{u} F_n = \left( \bar{u} A_v f_y w_{\text{eff,tie}} \right) / s_v \bar{F}_u \]  

(II-1-8)

where, \( \bar{u} = \) strength reduction factor of steel tie (= 0.75)

\( A_v \) = area of vertical shear reinforcement

\( f_y \) = yield strength of steel (= 414 MPa)

\( w_{\text{eff,tie}} \) = effective width of vertical tie (The half of the shear span is a default. A user can define the width in Assign-Outer Element)

\( \bar{F}_u \) = cross-sectional force of steel tie
1.3.4.3 Strength verification of struts

The strength condition of a concrete strut is verified by comparing the required width with provided width of the concrete strut, as shown below.

The strength conditions of concrete struts can also be verified visually as shown below.

Fig. II-1-9 Required/proposed area of concrete strut
1.3.4.4 Strength verification of nodal zones

The strength condition of a nodal zone is verified by comparing the required width with the provided width of the nodal zone boundary, as shown below.

1.3.5 Minimum rebars for crack control

Since the effective strength coefficient 0.75 was taken for the four inclined struts of the strut-tie model, the ACI 318M-14 requirement for minimum reinforcing bars for crack control must be satisfied.

\[ \sum \left( \frac{A_{si}}{b_{si}} \right) \sin \gamma_i \geq 0.003 \]

where \( A_{si} \) is the total area of distributed reinforcement at spacing \( s_i \) in the \( i \)-th direction of reinforcement crossing a strut at an angle \( \theta_i \) to the axis of a strut, and \( b_{si} \) is the width of the strut.
1.4 Design example - strut-tie model representing combined mechanism

1.4.1 Problem statement

In this section, the deep beam is designed by the strut-tie model representing the combined load transfer mechanism shown in Fig. II-1-1(c). As illustrated in the previous sections, the dimensions and the information on the bearing plates and loads are determined in the same way.

1.4.2 Construction of strut-tie model

The Beginning Mode is switched to the Modeling Mode to construct a strut-tie model for the deep beam. As the shear span-to-effective depth ratio of the deep beam is 1.27, the indeterminate strut-tie model that represents a combined load transfer mechanism is selected automatically from the template for deep beam. In the model, the top and bottom horizontal elements are placed 127 mm and 102 mm away from the top and bottom surfaces of the deep beam, respectively.

![Fig. II-1-10 Constructed strut-tie model](image-url)
1.4.3 Analysis of strut-tie model

The cross-sectional forces of struts and ties are determined by carrying out the finite element analysis of the constructed strut-tie model. As the model is an indeterminate truss structure, an iterative procedure is employed to find the converged axial rigidities and cross-sectional forces of struts and ties.

1.4.4 Strength verification and required rebars

1.4.4.1 Strength under bearing plates

The strength conditions under bearing plates are examined by the method illustrated in Section 1.2.4.1.
1.4.4.2 Required area of rebars

The required areas of main reinforcing bars are determined by the following equation. The requirement on the reinforcing bars is examined in the Design Review as shown below.

\[ A_{s, \text{req}} = \frac{F_u}{\phi f_y} \]  

(II-1-10)

where, \( \phi = \) strength reduction factor of steel tie (\( = 0.75 \))

\( F_u = \) cross-sectional force of steel tie

\( f_y = \) yield strength of steel (\( = 414 \text{ MPa} \))

The spacing of shear reinforcing bars is determined by the following equation.

\[ \bar{u} F_n = \left( \bar{u} A_v f_y w_{\text{eff,tie}} \right) / s, \bar{O} F_u \]  

(II-1-11)

where, \( \bar{u} = \) strength reduction factor of steel tie (\( = 0.75 \))

\( A_v = \) area of vertical shear reinforcement

\( f_y = \) yield strength of steel (\( = 414 \text{ MPa} \))

\( w_{\text{eff,tie}} = \) effective width of vertical tie (The half of the shear span is a default. A user can define the width in Assign-Outer Element)

\( F_u = \) cross-sectional force of steel tie
1.4.4.3 Strength verification of struts

The strength conditions of concrete struts are verified by comparing the required widths with provided (available) widths of concrete struts, as shown below.

The strength conditions of concrete struts can also be verified visually as shown below.
1.4.4.4 Strength verification of nodal zones

The strength condition of a nodal zone is verified by comparing the required width with the provided width of the nodal zone boundary, as shown below.
1.4.5 Minimum rebars for crack control

Since the effective strength coefficient 0.75 was taken for the six inclined struts of the strut-tie model, the ACI 318M-14 requirement for minimum reinforcing bars for crack control must be satisfied.

\[ \times (A_{si}/b_s) \sin \gamma_i \geq 0.003 \]  \hspace{1cm} (II-1-12)

where \( A_{si} \) is the total area of distributed reinforcement at spacing \( s_i \) in the \( i \)-th direction of reinforcement crossing a strut at an angle \( \gamma_i \) to the axis of a strut, and \( b_s \) is the width of the strut.

1.5 Summary

A simply supported reinforced concrete deep beam was designed by using two types strut-tie model. The amounts of flexural reinforcing bars are the same regardless of the model types. However, the strut-tie model representing a combined load transfer mechanism required the shear reinforcing bars additionally. As the design results can be different according to the strut-tie model, designers need to be cautious in the selection of strut-tie model. The deep beam template of AStrutTie helps users to select a most appropriate strut-tie model for deep beam by considering the shear span-to-effective depth ratio.